EECS 127/227AT Optimization Models in Engineering UC Berkeley Spring 2026

Discussion 14

1. Newton's Method

Consider the function $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = x^4. \tag{1}$$

(a) **Find the optimal value** $x^\star = \operatorname{argmin}_x f(x)$**.**

(b) Now, we analyze the performance of Newton's method on this problem. Starting from x_0, for $k \geq 0$ we take Newton steps of the form

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}. \tag{2}$$

Find the minimum number of Newton steps that are required to be within a distance of $\epsilon > 0$ **from optimum** $x^\star$**.** You may use the optimum value of $x^\star$ from part (a). Formally find $k^\star \in \mathbb{N}$ (where $\mathbb{N}$ is the set of positive integers) which is the smallest k for which $|x_k - x^\star| \leq \epsilon$, i.e.,

$$k^\star = \min_{k \in \mathbb{N} : |x_k - x^\star| \leq \epsilon} k. \tag{3}$$

Assume that $x_0 > \epsilon > 0$. Your answer should be in terms of ϵ and x_0.

2. Sphere Enclosure

Let B_i, $i = 1, \ldots, m$, be m Euclidean balls in $\mathbb{R}^n$, with centers $\vec{x}_i$, and radii $\rho_i \geq 0$. We wish to find a ball B with center $\vec{c} \in \mathbb{R}^n$ of minimum radius $r \geq 0$ that contains all the B_i, $i = 1, \ldots, m$. Cast this problem as an SOCP.